CONFINEMENT AND THE VORTEX VACUUM OF SU(2) LATTICE GAUGE THEORY

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The vortex theory which emerges from SU(2) lattice gauge theory by center projection is briefly reviewed. In this vortex picture, quark confinement is due to percolating (closed) vortices which are randomly linked to the Wilson loop. The deconfinement phase transition appears as a de-percolation phase transition.

1 Introductory remarks

Considering the nuclear force between hadrons as residuum of a confining force between the constituents of the hadrons explains why the understanding of quark confinement acquires highest priority in modern medium energy physics. In this context, lattice gauge theory [1] provides a convenient tool since it covers the non-perturbative aspects of quantum field theory which are important for the low energy regime. Numerical simulations of lattice QCD allow for a comparison of theoretical predictions with experimental data and trigger the interpretation of experimental data. Numerical simulations of pure SU(2) Yang-Mills are also very helpful to get insights into the basic mechanisms of gluon dynamics, which presumably dictates the structure of hadrons. The finite amount of computational power is the only limiting factor of the accuracy of the theoretical predictions. Modern computers allow for a number lattice points which corresponds to an ultra-violet cutoff of several GeVs by a size of the lattice universe of several fms. This size of the universe is several times of the size of the generic coherence length of the gluon sector, and the UV-cutoff is large enough to cover a wide span of low lying excitations.

Since lattice gauge simulations employ a discretized version of (Euclidean) space-time (with lattice spacing a), the crucial task is to extrapolate the lattice results to the continuum limit $a \to 0$. From continuum SU(2) gauge theory, we learn that the bare coupling constant g is a definite function of the UV regulator Λ. In fact, the self-consistent treatment of the gauge theory at high energies reveals [2]

$$\frac{1}{g^2(\Lambda)} = \frac{1}{g^2(\mu)} + \frac{11}{24\pi^2} \ln\left(\Lambda^2/\mu^2\right) , \qquad \text{(pure SU(2))} \tag{1}$$

where the prefactor of the logarithm depends on the gauge group and on the number of degrees of freedom. Keeping in mind that the UV cutoff in lattice simulations is provided by the lattice spacing, i.e. $\Lambda := \pi/a$, and defining $\beta := 4/g^2$, we conclude from (1) that

$$a^2(\beta) \;\propto\; \exp\left\{ -\frac{6\pi^2}{11}\,\beta \right\} \; , \qquad \text{for} \qquad \beta \gg 1 \; . \tag{2}$$

Any *physical* quantity which is measured in units of the lattice spacing a, e.g. σa^2 with σ the string tension, must show a unique dependence on β and exponentially decreases for large values of β. In particular, the ratio of two *physical* quantities, e.g. $m^2 a^2 / \sigma a^2$ with m a glue-ball mass, becomes independent of β for $\beta \gg 1$ (renormalization group invariance), and one safely extrapolates the mass squared in units of the string tension to the continuum limit $a \to 0$. Due to β–independence, the string tension becomes the only parameter of pure Yang-Mills theory (dimensional transmutation).

In the recent past, many observables have been addressed in computer simulations. For a review see [3]. Besides these "lattice measurement" of observables, many efforts were devoted to support or to invalidate scenarios of quark confinement. Among the promising ideas, I would like to mention the scenario of the dual super-conductor[4] which assumes a condensation of Abelian monopoles which then yields a dual Meissner effect. Over the last two decades, the evidence has increased that a vortex type structure of the vacuum is responsible for quark confinement [5,6,7,8,9]. In this paper, we will further pursue the vortex picture and will study its properties at finite temperatures.

2 Towards the roots of confinement

The fundamental degrees of freedom of pure SU(2) lattice Yang-Mills theory are SU(2) matrices $U(b)$ which are defined at the links b of the lattice. The partition function is a functional integral over these matrices, i.e.

$$Z \;=\; \int \mathcal{D}U \;\exp\{-S_W\} \; , \qquad S_W \;=\; \beta \sum_p \left[1 - \mathcal{P}(p)\right] \; , \tag{3}$$

where $\mathcal{P} := \prod_{b\in p} U(b)$ is defined at the plaquette p of the lattice and $\mathcal{D}U$ includes the Haar measure. Despite the discretization of space-time, action and partition function enjoy an exact gauge invariance $U(b) \to \Omega(x)U(b)$, $\Omega \in SU(2)$. The Wilson action S_w reduces to the standard action of continuum Yang-Mills theory in the naive continuum limit $a \to 0$. Other choices are

possible and differ from S_w by *irrelevant terms* . These irrelevant terms can be either chosen to improve the convergence towards the continuum limit [10] or to suppress the statistical noise [11].

In this paper, we will focus on the mechanism of quark confinement. The potential $V(r)$ of two static quarks, located at distance r, can be obtained from the Wilson loop $\mathcal{W}$

$$\mathcal{W} := \prod_{b \in \mathcal{C}} U(b) \; , \qquad \langle \mathcal{W} \rangle \; \propto \; \exp\{-V(r)T\} \qquad (T \to \infty) \; , \tag{4}$$

where $\mathcal{C}$ is a rectangular of spatial extension r and extension T in time direction. The so-called *area law* , i.e. $\langle \mathcal{W} \rangle \propto \exp\{-\sigma \mathcal{A}\}$ with $\mathcal{A}$ the minimal area enclosed by $\mathcal{C}$, is considered as confinement criterium, since one observes a linear rising confining potential $V(r) = \sigma \, r$ in this case.

In a recent important work [8], the authors introduced the so-called *center projection*, which considerably reduces the number of degrees of freedom, while it preserves those relevant for confinement. Center projection firstly exploits the gauge degree of freedom for maximizing $\sum_b \{\mathrm{tr} U(b)\}^2$. After this gauge fixing, the link variables $U(b)$ are as close to ± 1 as possible. Secondly, one projects $U(b)$ onto the corresponding Z_2 variable $\mathcal{U}(b)$. One easily checks that the particular gauge invariance provided by $\Omega(x) \in Z_2$ is not affected by the projection technique. This implies that this technique induces a Z_2 gauge theory.

It turns out that the off-diagonal elements of the link matrices of a particular lattice configuration are generically not small implying that the (relative) error induced into a generic observable by projection can reach 60% [9]. The crucial observation is that, by contrast, the string tension is almost unchanged [8,9] (center dominance). This bears the conjecture that the induced Z_2 gauge theory still contains the degrees of freedom relevant for confinement. In the very recent past, many results support this idea [12].

The Z_2 gauge theory can be considered as a vortex theory. One says that a vortex pierces the plaquette p if $v(p) := \prod_{b \in p} \mathcal{U}(b) = -1$. In order for revealing the string type nature of the vortices for a given time slice, we consider the plaquettes constituting a cube c, part of the spatial hypercube of space-time, and find $\prod_{p \in c} v(p) = 1$. This implies that the number of vortices piercing the plaquettes of the cube c must be even. For this reason, the vortices necessarily form closed lines in space.

Are these vortices lattice artifacts or do they survive the continuum limit? In order for answering this question, we investigated the vortex (area) density ρ which counts the average number of vortices piercing an area element [9]. The important observation is that ρa^2 exhibits the characteristic dependence on β (see (2)) signaling that the vortices are physical objects rather than lattice artifatcs. We roughly find $\rho \approx 2\,\mathrm{fm}^{-2}$. We subsequently studied the vortex interactions by calculating the correlations between points where vortices intersect the plane [13].It turns out that this interaction also shows the desired renormalization group behavior attesting physical relevance. The vortex interaction is medium range attractive and possesses a range of $\approx 0.4\,\mathrm{fm}$ (see [13]).

Let us neglect the inter vortex correlations for getting an idea of the confinement mechanism in the vortex picture. The expectation value of the Wilson loop is in this case [13]

$$\langle \mathcal{W} \rangle \;=\; \langle \prod_{b \in \mathcal{C}} \mathcal{U}(b) \rangle \;=\; \langle \prod_{p \in \mathcal{A}} v(p) \rangle \;=\; \sum_n (-1)^n \, P(n) \; , \tag{5}$$

where $\mathcal{C} = \partial \mathcal{A}$ and $P(n)$ is the probability of finding n vortices which are linked to $\mathcal{C}$. The sum in (5) can be easily evaluated for the case of the random vortex model [13]. One recovers the desired area law $\langle \mathcal{W} \rangle = \exp\{-2\rho \mathcal{A}\}$. The string tension is $\sigma_{rand} = 2\rho \approx (400\,\mathrm{MeV})^2$, which is in good agreement with the exact value $\sigma = (440\,\mathrm{MeV})^2$.

3 Nature of the deconfinement phase transition

Once we have evolved a definite scenario of the confinement mechanism in the vortex picture (last section), the important question is whether the vortex vacuum correctly accounts for the deconfinement phase transition at finite temperature. It is known for a long time that SU(2) Yang-Mills theory undergoes a deconfinement phase transition at a critical temperature T_c. Choosing $\sigma = (440\,\mathrm{MeV})^2$ as reference scale, one finds $T_c \approx 210\,\mathrm{MeV}$ for a pure SU(2) gauge theory, and $T_c \approx 150\,\mathrm{MeV}$ for the realistic case of lattice QCD [14].

Our numerical simulations show that, for a pure SU(2) gauge theory, the heavy quark potential is almost unchanged by center projection even at finite temperatures [15]. In particular, the correct transition temperature T_c is found resorting to the induced Z_2 gauge theory.

For revealing temperature effects, it is convenient to distinguish Wilson loops lying in a tx-plane from Wilson loops which are embedded in the spatial hypercube. in the following, we therefore contrast the time-like string tension

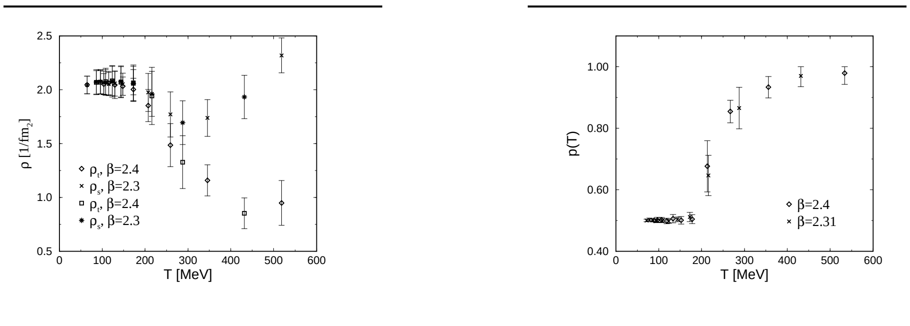

Figure 1: The timelike and spatial vortex area densities (left) and the vortex pairing (right) as function of temperature.

to the spatial string tension. One should, however, keep in mind that the spatial string tension lacks a direct physical interpretation. In order for gaining definite insights into the nature of the deconfinement phase transition, we studied time-like and spatial vortex (area) densities as function of temperature, since these densities sets the scale of the corresponding string tensions. The result is shown in figure 1. As expected, ρ_s and ρ_t coincide for $T < T_c$. For $T > T_c$, ρ_t decreases, whereas ρ_s is slightly suppressed at $T \approx T_c$ and strongly increases for $T \gg T_c$. The sum of ρ_t and ρ_s roughly stays constant. This indicates that the vortices get polarized along the time axis direction. However, these polarization cannot explain the sharp deconfinement phase transition, since ρ_t has only dropped by a factor of 2 at $T \approx 2T_c$. An inspection of (5) shows that the string tension would vanish if the number n of vortices which pierce a given area is even. The quantity relevant for the phase transition is therefore the vortex pairing

$$p(T) \; = \; \frac{\langle N_{even} \rangle}{\langle N_{even} + N_{odd} \rangle} \; , \tag{6}$$

where $N_{even(odd)}$ counts the events that an even (odd) number of vortices of a given MC configuration was linked to the Wilson loop. The numerical result for $p(T)$ is also presented in figure 1. It shows a clear signal of the phase transition at $T = T_c$ (see [15]).

The numerical results, presented above, are consistent with the following vortex picture: at low temperatures, i.e. $T < T_c$, the vortices *percolate*. Following the vortex network, one can "travel" across the whole lattice universe. The infinite vortex cluster size provides the long range correlations which are

imperative for observing the area law for large size Wilson loops. For $T > T_c$, the vortices stop percolating. In this case, the vortices form small size loops, which hardly intersect. Due to the small sizes of the vortex clusters, only vortex clusters which are located close to the perimeter of the Wilson loop can contribute a non-trivial factor, i.e. $(-1)^n$, n odd, and the de–percolation of the vortices at $T = T_c$ results in a perimeter law of the Wilson loop. In the vortex picture, the deconfinement phase transition appears as a de-percolation transition. Our most recent lattice calculations confirm this scenario [16]: the probability distribution of the maximum sizes of the vortex clusters reveals a clear signal of de-percolation at $T = T_c$.

4 Conclusions

As already mentioned by 't Hooft as long as twenty years ago, particular gauges might be more convenient than others for providing a definite picture of quark confinement. The so-called Abelian gauge support the intuitive picture of the dual Meissner effect [4]. Recent work [8] proposes the so-called center gauge fixing and provides a definite prescription to project the SU(2) lattice Yang-Mills theory onto a Z_2 gauge theory. The latter theory offers a precise definition of a vortex theory, which is similar to those which were available in the literature for quite some time [5,6,7]. The vortex theory arising from center projection turned out to be very fruitful for understanding quark confinement. Let me summarize the most important results: the vortex theory reproduces the string tension within the statistical errors [8,9] although the generic (relative) error produced by center projection is expected to be $\approx 60\,\%$ [9]. The vortex properties, such as the vortex area density or the inter vortex correlation length, show the correct renormalization group behavior [9,13]. The vortex picture extrapolates to the continuum limit. The vortices are physical objects rather than lattice artifacts. Zero temperature quark confinement is due to vortices which are randomly linked to the Wilson loop. The random distribution of the vortices is thereby due to vortex percolation [15,16]. The concept of vortex dominance of the string tension also extends to finite temperatures [15]. The deconfinement transition temperature is recovered in the effective Z_2 gauge theory to high accuracy. The deconfinement phase transition can be understood in the vortex theory as a de-percolation transition.

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References

1. H. J. Rothe, *Lattice Gauge Theories*, World Scientific Lecture Notes in Physics, Vol.43, 1992.
2. F. J. Yndurain, *Quantum Chromodynamics* , Springer Verlag, 1983.
3. JLQCD Collaboration, *Phys. Rev. Lett.* **80**, 5711 (1998); UKQCD Collaboration, *Phys. Rev.* D **54**, 3619 (1996).
4. S. Mandelstam, *Phys. Rep.* C **23**, 245 (1976); G. 't Hooft, *Nucl. Phys.* B **190**, 455 (1981).
5. G. Mack and E. Pietarinen, *Nucl. Phys.* B **205**, 141 (1982);
6. J. M. Cornwall, *Nucl. Phys.* B **157**, 392 (1979); *Phys. Rev.* D **57**, 7589 (1998).
7. E. T. Tomboulis, Phys. Lett. **B303** (1993) 103; T. G. Kovács and E. T. Tomboulis, *Phys. Rev.* D **D57**, 4054 (1998).
8. L. Del Debbio, M. Faber, J. Greensite and Š. Olejník, Phys. Rev. **D 55** (1997) 2298.
9. K. Langfeld, H. Reinhardt, O. Tennert, *Phys. Lett.* B **419**, 317 (1998).
10. P. Hasenfratz, F. Niedermayer, *Nucl. Phys.* B **414**, 785 (1994); W. Bietenholz, U. J. Wiese, *Nucl. Phys.* B **464**, 319 (1996).
11. K. Langfeld, H. Reinhardt, O. Tennert, *Phys. Rev.* D **56**, 6798 (1997).
12. L. Del Debbio, M. Faber, J. Giedt, J. Greensite and Š. Olejník, hep-lat/9801027.
13. M. Engelhardt, K. Langfeld, H. Reinhardt and O. Tennert, *Phys. Lett.* B **431**, 141 (1998).
14. K. Kanaya, Lattice Field Theory, Melbourne, Australia, 11-15 Jul 1995, Nucl. Phys. Proc. Suppl. **47**, 144 (1996).
15. K. Langfeld, H. Reinhardt, M. Engelhardt, O. Tennert, hep-lat/9805002.
16. M. Engelhardt, K. Langfeld, H. Reinhardt and O. Tennert, in preparation.